

# The Effect of Altitude on Small Wind Turbine Power Production,

## or How to Estimate Power Production if You Don't Own Beach-Front Property

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Electricity generated, typically measured in kilowatt hours (kWh), is the most important factor in determining the economic effectiveness of a small wind turbine. A reliable estimate of expected production is therefore critical to assessing wind turbine potential at your site. Unfortunately, estimating wind production can be challenging because it depends on many factors unique to each situation. Many Web sites and articles provide generally acceptable methods to simplify production estimates. These simplified approaches often provide reasonable production estimates for preliminary analyses. All of the simplified methods, however, depend on a set of assumptions that may differ significantly from individual situations in Wyoming. One factor that makes small wind turbines in Wyoming unique is altitude. Most simple methods to estimate production rely on assumptions relevant for sea level sites, but Wyoming is far from sea level (the lowest elevation in Wyoming is 3,099 feet above sea level). In this bulletin, we discuss the impact of altitude on small-wind turbine ( $\leq 25$  kilowatts rated capacity) production and make recommendations for the homeowner considering a small wind turbine who doesn't live on a beach.

The amount of power, measured in watts or kilowatts (1 kilowatt = 1,000 watts), a turbine can generate is a function of many site-specific factors. Most important are the average annual wind speed and rotor swept area; however, air density also affects power and air density changes with al-

titude (and temperature). At sea level (and 68 degrees F) air density is approximately 0.076 lb/ft<sup>3</sup>. Air density drops with altitude – to 0.069 lb/ft<sup>3</sup> at 3,500 ft, to 0.062 lb/ft<sup>3</sup> at 7,000 feet, and to 0.056 lb/ft<sup>3</sup> at 10,000 feet (Figure 1).

Why should Wyoming homeowners worry about the elevation-air density relationship? Many commonly available wind turbine production formulas appear to use sea level air

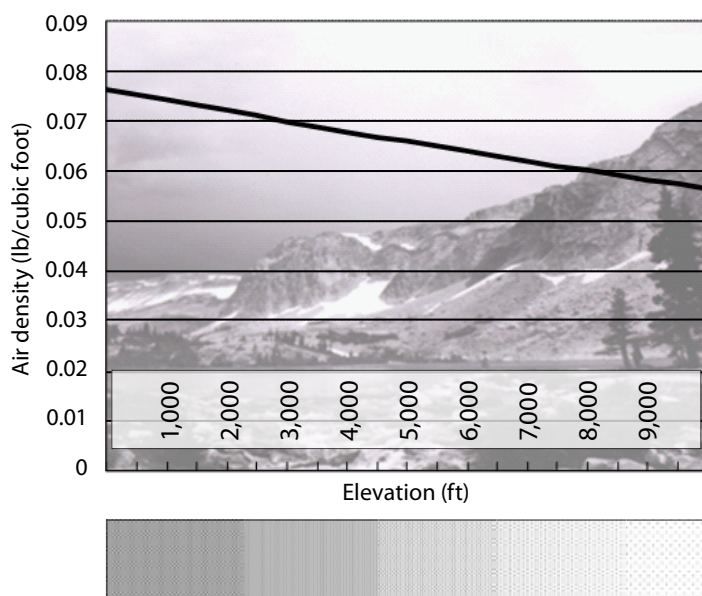


Figure 1. Relationship between elevation and air density

density (sometimes it is hard to tell). For relatively low elevations (< 3,000 feet), the change in air density will not matter much. As elevation increases, the decrease in air density can meaningfully affect power production estimates (Table 1). The predicted annual energy output (AEO), measured in kWh/yr, is approximately 10-percent lower at 3,500 feet than at sea level (holding everything else constant). At 7,000 feet, production from a small wind turbine could be nearly 20-percent lower than the same turbine at sea level. The change in energy production at altitude can have significant impacts on the economic feasibility of residential wind turbines.

**Table 1. Relationship between Altitude, Air Density, and Annual Energy Output (AEO)<sup>a</sup>**

Altitude	Air Density (lb/ft <sup>3</sup> )	Power (kW)	AEO (kWh/yr)	% Difference from Sea Level
0	0.076	0.298	2,612	
500	0.075	0.294	2,574	-1.5%
1,000	0.074	0.289	2,536	-2.9%
1,500	0.073	0.285	2,499	-4.3%
2,000	0.072	0.281	2,462	-5.7%
2,500	0.071	0.277	2,426	-7.1%
3,000	0.070	0.273	2,390	-8.5%
3,500	0.069	0.269	2,354	-9.8%
4,000	0.068	0.265	2,319	-11.2%
4,500	0.067	0.261	2,285	-12.5%
5,000	0.066	0.257	2,250	-13.8%
5,500	0.065	0.253	2,216	-15.1%
6,000	0.064	0.249	2,183	-16.4%
6,500	0.063	0.245	2,150	-17.7%
7,000	0.062	0.242	2,117	-18.9%
7,500	0.061	0.238	2,085	-20.2%
8,000	0.060	0.234	2,053	-21.4%
8,500	0.059	0.231	2,021	-22.6%
9,000	0.058	0.227	1,990	-23.8%
9,500	0.057	0.224	1,959	-25.0%
10,000	0.056	0.220	1,929	-26.2%

<sup>a</sup> Assumes a turbine with a 12-foot rotor diameter, average annual wind speeds of 12 mph, and a turbine power coefficient of 0.3. The percentage differences from sea level are not sensitive to these assumptions.

<sup>1</sup> Note: This approach grows more conservative as elevation increases because the altitude-air density relationship is not quite linear (air density decreases at a decreasing rate).

Lower power production means lower energy savings – a turbine at sea level is more likely to make economic sense than an identical turbine in identical conditions at higher altitude. Thus, if you live above 3,500 feet and use a formula designed for sea level, you will overestimate production. This can make a wind application look like a better deal than it really is. For example, the simple payback (the number of years it takes for the turbine investment to pay for itself) for identical systems is nearly 14-percent longer at 5,000 feet than it is at sea level. Thus, a system predicted to reach payback in year 15 at sea level will likely take two to three years longer to reach payback at 5,000 feet.

So, how to estimate expected power production if you are considering a wind turbine at high altitude? Before reading further, remember that the altitude effect is very small relative to the effect of wind speeds on production. If you are going to get one variable right, make it wind speed! But if you already have an accurate wind speed estimate, there are a few simple adjustments you can make to adjust your power estimates for altitude.

First, for a quick estimate of the elevation effect, simply use a readily available sea level formula to estimate power production, and then subtract 1.4 percent for every 500 feet above sea level<sup>1</sup>. For example, suppose the sea level formula estimates that a given turbine at your location will produce 4,500 kWh/yr. If you live at 6,000 feet, subtract 756 kWh/yr to get an altitude-adjusted estimate of 3,744 kWh/yr:  $(6,000/500) \times (4,500 \times 0.014) = 756$ .

If a sea level energy estimate is not available, estimate power production from scratch using this simple annual energy output (AEO) formula:

$$\text{AEO (kWh/yr)} = k \times D^2 \times V^3,$$

where D is the rotor diameter in feet, V is the average annual wind speed in mph, and k is a constant that accounts for conversion factors, the power coefficient, and air density. You can approximate the altitude-air density effect by adjusting the constant using Table 2.

**Table 2. Altitude Adjusted Constant for Simple Annual Energy Output Formula**

Altitude (ft)	Constant (k) <sup>a</sup>
3,000 – 3,999	0.0079
4,000 – 4,999	0.0077
5,000 – 5,999	0.0075
6,000 – 6,999	0.0073
7,000 – 7,999	0.0070
8,000 – 8,999	0.0068
9,000 +	0.0066

<sup>a</sup> Assumes a conservative power coefficient of 0.25.

For example, if you live at 6,000 feet you can estimate annual energy output with AEO (kWh/yr) = 0.0073 × D<sup>2</sup> × V<sup>3</sup>. Thus, an average wind speed of 12 mph and a rotor diameter of 12 feet would be expected to produce: 0.0073 × (12)<sup>2</sup> × (12)<sup>3</sup> = 1,817 kWh/yr. This approach will generally be within 1 percent of estimates made with more sophisticated formulas. It depends, however, on assuming a constant power coefficient of 0.25. Power coefficients actually vary significantly across different turbines and wind speeds.

A more careful calculation can be made using the following AEO formula:

$$\text{AEO (kWh/yr)} = K \times C_p \times D^2 \times V^3,$$

where

$K = 0.4574$  is a constant to convert metric to standard units and output to kilowatt hours per year,

$C_p$  = the power coefficient (i.e., the percentage of available wind power captured by the turbine), which typically is assumed to range between 0.25 and 0.45 (0.59 is the theoretical maximum power coefficient),

$\rho$  = air density (lb/ft<sup>3</sup>),

$D$  = rotor diameter in feet, and

$V$  = average annual wind speed (mph).

Once the appropriate values for the power coefficient ( $C_p$ ), rotor diameter ( $D$ ) and wind speed ( $V$ ) are determined, use the air density value from Table 1 most consistent with elevation at your site.

Knowing the amount of power in your wind resource is critical to making a sound decision about installing a small wind turbine. Altitude is only one of the many factors (and a relatively small factor) affecting your wind power potential, but accounting for altitude will help ensure you make the most informed decision for your location. For additional help understanding your wind resource, please contact your local University of Wyoming Cooperative Extension Service office or visit us on the Web ([www.ces.uwyo.edu](http://www.ces.uwyo.edu)).

### Appendix: Altitude-Air Density Calculations (SI Units)

We estimate the relationship between altitude and air density using values from the International Standard Atmosphere model. Air density at different altitudes depends on temperature and pressure. Temperature ( $T$ ) at altitude  $x$  meters is given by:

$$T = T_0 - L \times x,$$

where

$T_0 = 288.15$  K is the sea level standard atmospheric pressure, and

$L = 0.0065$  K/m is the temperature lapse rate.

The pressure ( $p$ ) at altitude  $x$  is given by:

$$p = p_0 \times \left( 1 - \frac{L \times x}{T_0} \right)^{\frac{g \times M}{R \times L}}$$

where

$p_0 = 101325$  Pa is the sea level standard atmospheric pressure,

$g = 9.80665$  m/s<sup>2</sup> is earth surface gravitational acceleration,

$M = 0.0289644$  is the molar mass of dry air, and

$R = 8.31447$  J/(mol×K) is the universal gas constant.

Then, air density ( $\rho$ ) at altitude  $x$  is given by:

$$\rho = \frac{p \times M}{R \times T}.$$

### Sources:

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